Question 10 (9 marks)

(a) Draw the subset of the complex plane determined by on the axes below.

(3 marks)

<EFOFEX>
id:fxd{83c81f21-553b-46d0-bb68-ea3effab2e03}

FXData:

</EFOFEX>

<EFOFEX>
id:fxd{3b95f65c-8585-4808-a8f7-0c5259a934c2}

FXData:

</EFOFEX>

(b) The circular arc in the diagram represents  
the locus of a complex number .  
  
  
Without using or , write  
equations or inequalities in terms of   
for the indicated locus.  
  
  
  
 (3 marks)

(c) Describe the subset of the complex plane determined by .

(3 marks)

Question 16 (8 marks)

(a) Determine all solutions to the equation in exact polar form. (3 marks)

(b) Consider the ninth roots of unity expressed in polar form .

(i) Determine the roots for which . (2 marks)

(ii) Use all nine roots to show that .

(3 marks)

Question 16 (8 marks)

(a) Determine all solutions to the equation in exact polar form. (3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ expresses in polar form  ü states one correct solution  ü states all correct solutions |

(b) Consider the ninth roots of unity expressed in polar form .

(i) Determine the roots for which . (2 marks)

|  |
| --- |
| Solution |
| Hence |
| Specific behaviours |
| ✓ general expression for roots  ü correct roots |

(ii) Use all ten roots to show that .

(3 marks)

|  |
| --- |
| Solution |
| The nine roots are given by , and the sum of these roots, and hence their real parts, will be :  But and . Hence |
| Specific behaviours |
| ✓ uses sum of real parts of all roots is  ü uses and known values  ü simplifies to obtain required result |

Question 9 (9 marks)

<EFOFEX>
id:fxd{d1a88875-eb2f-4c7d-ac86-281b86479acb}

FXData:
</EFOFEX>The complex numbers and are  
shown in the complex plane at right.

(a) Express in polar form. (1 mark)

(b) Express in Cartesian form. (1 mark)

(c) Plot in the complex plane above, given that . (2 marks)

(d) Determine the argument of when . (2 marks)

(e) Let . Express in polar form in terms of the real constants and .

(3 marks)

Question 9 (9 marks)

<EFOFEX>
id:fxd{27e903bb-1d18-41ee-9768-6c2382c3816d}

FXData:

</EFOFEX>The complex numbers and are  
shown in the complex plane at right.

|  |
| --- |
| Solution (c) |
| See diagram for |
| Specific behaviours |
| ✓ correct magnitude  ü correct argument |

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct expression |

(a) Express in polar form. (1 mark)

(b) Express in Cartesian form. (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct expression |

(c) Plot in the complex plane above, given that . (2 marks)

(d) Determine the argument of when . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates one correct argument of factors  ü correct argument |

(e) Let . Express in polar form in terms of the real constants and .

(3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ expresses all terms in polar form  ü correct magnitude in terms of  ü correct argument in terms of |

Question 19 (9 marks)

(a) Plot the complex number that satisfies the conditions and on the Argand diagram below. (2 marks)

<EFOFEX>
id:fxd{aecc4253-2e5b-4722-9a70-db3592a96c42}

FXData:

</EFOFEX>

(b) Let and be another complex number. The locus of a complex number satisfies the condition and is shown in the diagram below.

<EFOFEX>
id:fxd{4c0be6a6-ead6-49b1-8fd0-59bc376d970c}

FXData:

</EFOFEX>

(i) Determine the complex number . (2 marks)

(ii) On the same diagram, indicate the locus of a complex number that satisfies the condition . (1 mark)

(c) The locus of points that satisfy is an arc of a circle.

(i) Sketch the locus of in the complex plane. (2 marks)

<EFOFEX>
id:fxd{928e6a15-a0f9-4894-9659-4cf91259c7ae}

FXData:

</EFOFEX>

(ii) Determine, with justification, the exact location of the centre of the circle. (2 marks)

Question 19 (9 marks)

(a) Plot the complex number that satisfies the conditions and on the Argand diagram below. (2 marks)

<EFOFEX>
id:fxd{24f65691-ef6a-46f2-bd66-b90c0897e2a9}

FXData:

</EFOFEX>

|  |
| --- |
| Solution |
| See diagram |
| Specific behaviours |
| ✓ correctly indicates at least one ray  ü correctly plots complex number |

(b) Let and be another complex number. The locus of a complex number satisfies the condition and is shown in the diagram below.

<EFOFEX>
id:fxd{a9e877f4-07d4-44ce-9f16-ef33c6c53768}

FXData:

</EFOFEX>

|  |
| --- |
| Solution (b)(ii) |
| See shading on diagram |
| Specific behaviours |
| ✓ correct shading |

(i) Determine the complex number . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates point lies on perpendicular to locus through  ü correct complex number |

(ii) On the same diagram, indicate the locus of a complex number that satisfies the condition . (1 mark)

(c) The locus of points that satisfy is an arc of a circle.

(i) Sketch the locus of in the complex plane. (2 marks)

<EFOFEX>
id:fxd{c32d4d29-1ad9-4f9c-8582-8bbb8c24334b}

FXData:

</EFOFEX>

|  |
| --- |
| Solution |
| Anticlockwise major arc from to .  *NB Marks for location of major arc rather than neatness/curvature* |
| Specific behaviours |
| ✓ major arc of a circle drawn anywhere  ü correctly locates endpoints and major arc drawn to their right |

(ii) Determine, with justification, the exact location of the centre of the circle. (2 marks)

|  |
| --- |
| Solution |
| When , then and a right-triangle is formed in the circle. The midpoint of the hypotenuse of this triangle must be the centre of the circle. Hence the centre is at |
| Specific behaviours |
| ✓ indicates adoption of suitable method  ü correct centre, fully justified |

Question 12 (8 marks)

Consider the complex number .

(a) Let be the subset of the complex plane that satisfies .

(i) Sketch the subset . (3 marks)

<EFOFEX>
id:fxd{fdfbb888-2d0a-415c-a323-d78a58007188}

FXData:

</EFOFEX>

(ii) Determine the exact minimum value of in . (2 marks)

(b) Sketch the subset of the complex plane that satisfies the equation .

(3 marks)

<EFOFEX>
id:fxd{cc18cbfe-a099-43c8-9332-18744ff6c491}

FXData:

</EFOFEX>

Question 12 (8 marks)

Consider the complex number .

(a) Let be the subset of the complex plane that satisfies .

(i) Sketch the subset . (3 marks)

<EFOFEX>
id:fxd{aed90142-003d-4c82-b695-f02f2e9ba6b8}

FXData:

</EFOFEX>

|  |
| --- |
| Solution |
| . See diagram. |
| Specific behaviours |
| ✓ indicates two given points  ü draws their perpendicular bisector  ü indicates correct side of line |

(ii) Determine the exact minimum value of in . (2 marks)

|  |
| --- |
| Solution |
| Require minimum distance from to . |
| Specific behaviours |
| ✓ indicates value as distance  ü correctly calculates distance |

(b) Sketch the subset of the complex plane that satisfies the equation .

(3 marks)

<EFOFEX>
id:fxd{e0349977-b96a-45d0-8ffc-ed9d71ec876d}

FXData:

</EFOFEX>

|  |
| --- |
| Solution |
| Arguments of from and must be opposite. |
| Specific behaviours |
| ✓ indicates arguments from and opposite  ü correct rays in complex plane  ü indicates ray ends not included |

Question 16 (7 marks)

Let be a rectangle in the complex plane, where is the origin. The points and represent the complex numbers and respectively, where and .

(a) Draw a labelled sketch of the rectangle in the complex plane. (2 marks)

<EFOFEX>
id:fxd{85b32429-2f46-444f-91f0-7f217ad566f4}

FXData:

</EFOFEX>

(b) Determine the complex number represented by . (1 mark)

Rectangle is rotated about in an anticlockwise direction to .

(c) Determine in exact Cartesian form the complex numbers represented by the points and . (4 marks)

Question 16 (7 marks)

Let be a rectangle in the complex plane, where is the origin. The points and represent the complex numbers and respectively, where and .

(a) Draw a labelled sketch of the rectangle in the complex plane. (2 marks)

<EFOFEX>
id:fxd{d452f566-0c24-4c82-b65d-91844082ea32}

FXData:

</EFOFEX>

|  |
| --- |
| Solution |
| See diagram |
| Specific behaviours |
| ✓ rectangle, in fourth quadrant  ü in first quadrant, |

(b) Determine the complex number represented by . (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ü correct complex number for |

Rectangle is rotated about in an anticlockwise direction to .

(c) Determine in exact Cartesian form the complex numbers represented by the points and . (4 marks)

|  |
| --- |
| Solution |
| Let so that multiplying any complex number by will rotate it anticlockwise in the complex plane. Then |
| Specific behaviours |
| ✓ indicates use of multiplication by complex number for rotation  ü complex number for  ü simplified complex number for  ü simplified complex number for |

Question 17 (7 marks)

(a) Let the complex numbers and . Determine the modulus and argument of in terms of the real constants and .

(2 marks)

(b) Let the complex numbers and . Determine the value(s) of the real constant given that is purely imaginary. (2 marks)

(c) The complex number is a root of the equation ,  
where and are real non-zero constants.  
  
Determine two roots of the complex equation . (3 marks)

Question 17 (7 marks)

(a) Let the complex numbers and . Determine the modulus and argument of in terms of the real constants and .

(2 marks)

|  |
| --- |
| Solution |
| Hence and . |
| Specific behaviours |
| ✓ modulus  ü argument |

(b) Let the complex numbers and . Determine the value(s) of the real constant given that is purely imaginary. (2 marks)

|  |
| --- |
| Solution |
| . |
| Specific behaviours |
| ✓ expresses in real and imaginary parts  ü correct values of |

(c) The complex number is a root of the equation ,  
where and are real non-zero constants.  
  
Determine two roots of the complex equation . (3 marks)

|  |
| --- |
| Solution |
| is a solution, so require square roots of . |
| Specific behaviours |
| ✓ indicates square roots of required  ü one root  ü second root |

Question 9 (6 marks)

Let and be complex numbers, and and be constants so that

Determine the following in terms of and / or .

(a) . (3 marks)

(b) . (3 marks)

Question 9 (6 marks)

Let and be complex numbers, and and be constants so that

Determine the following in terms of and / or .

(a) . (3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates correct arguments for and  ü uses difference of arguments for quotient  ü uses sum of arguments for product and simplifies |

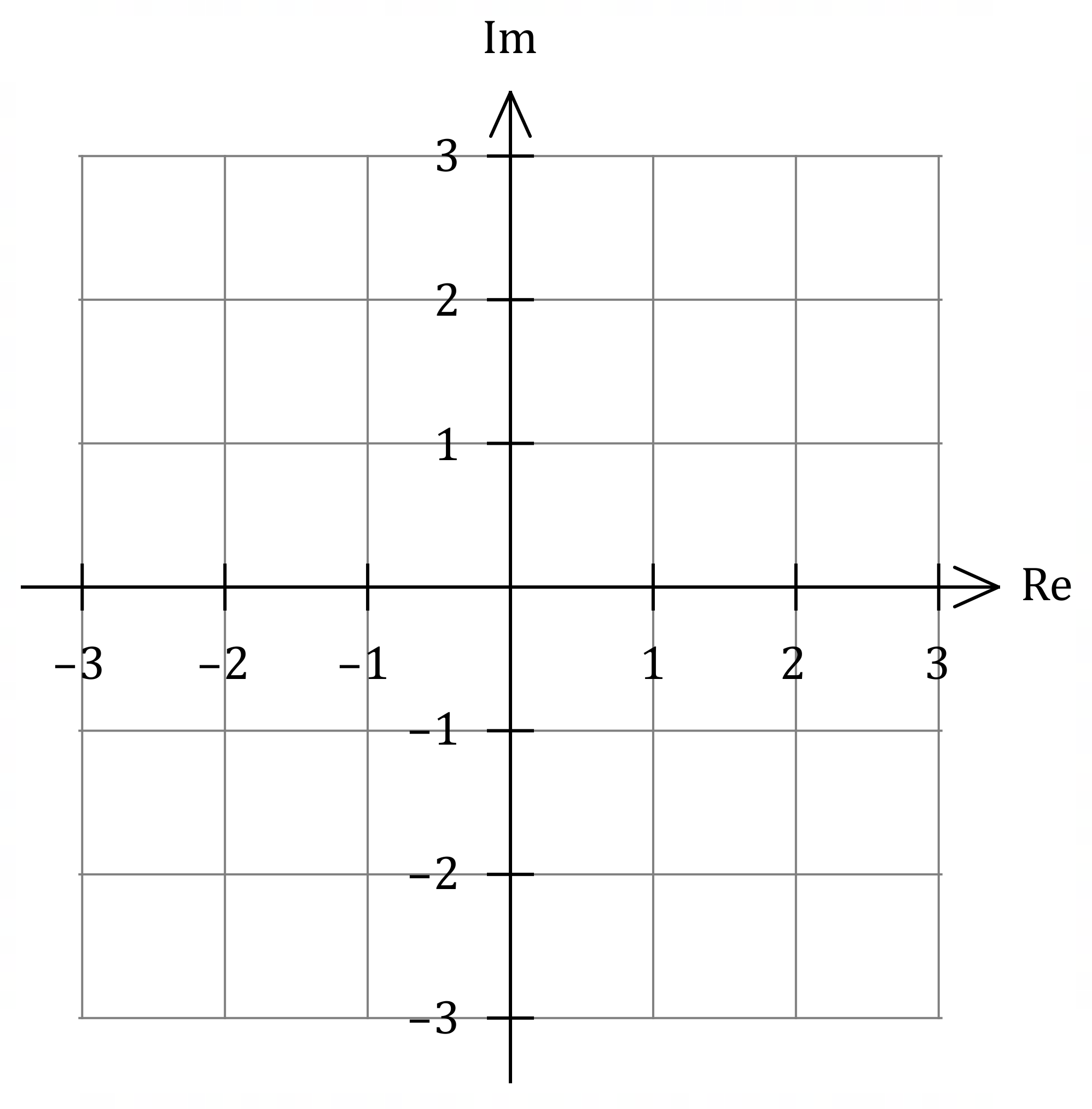
(b) . (3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates correct arguments for and  ü correct modulus for  ü correct modulus for |

Question 10 (7 marks)

Let be the region of the complex plane where the inequalities and hold simultaneously.

(a) Sketch on the axes below. (5 marks)

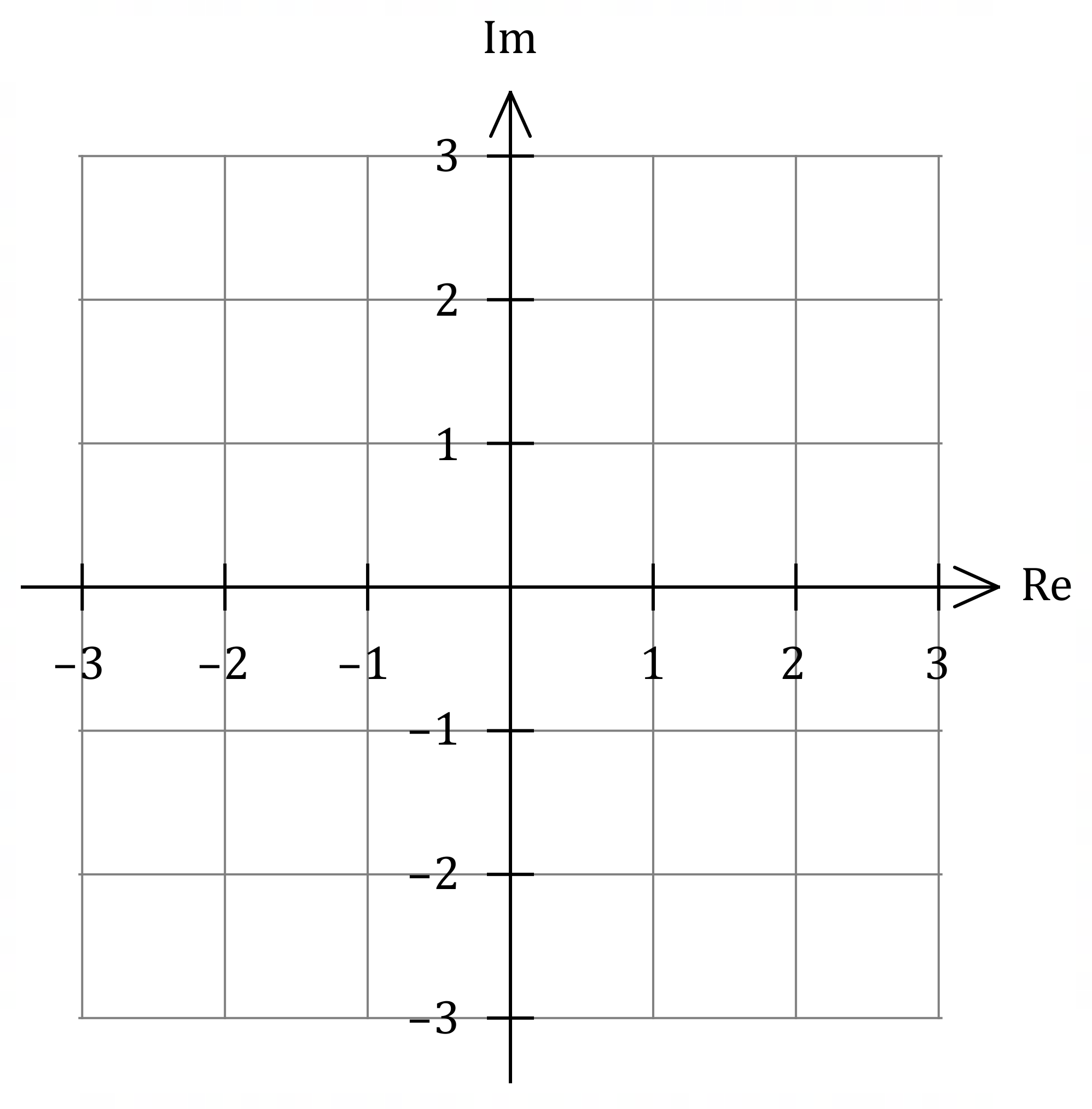


(b) Determine the maximum value of in . (2 marks)

Question 10 (7 marks)

Let be the region of the complex plane where the inequalities and hold simultaneously.

(a) Sketch on the axes below. (5 marks)

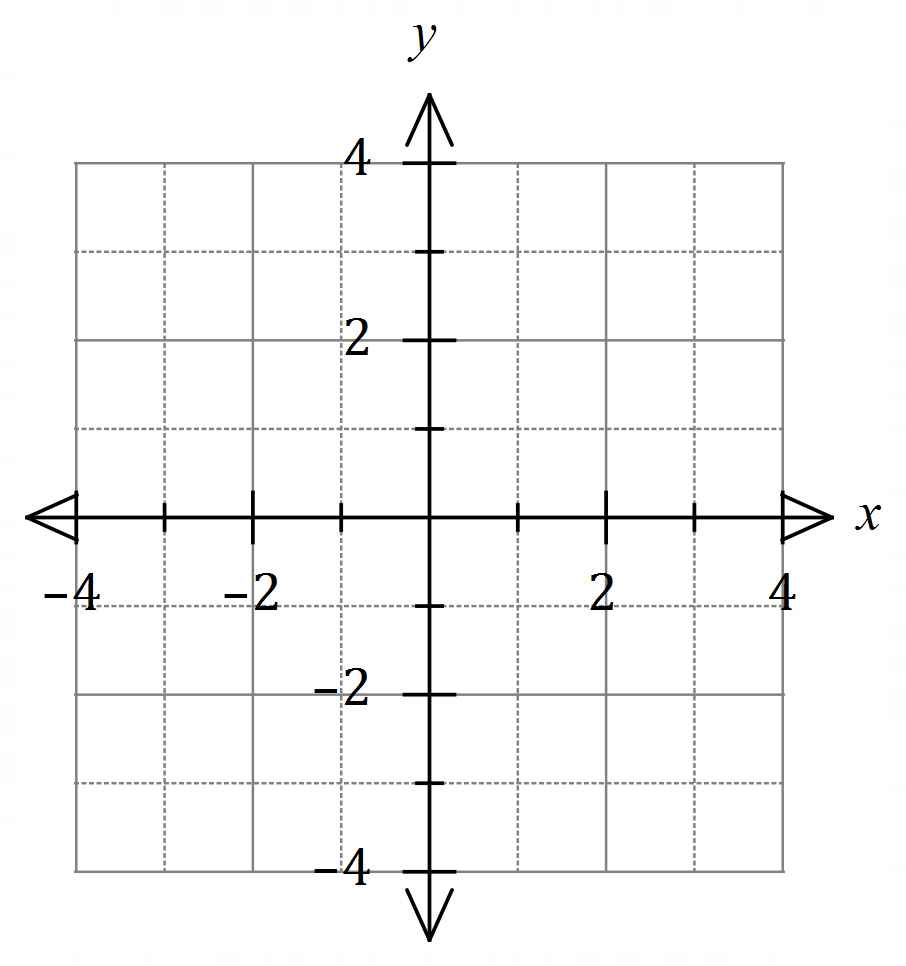


(b) Determine the maximum value of in . (2 marks)

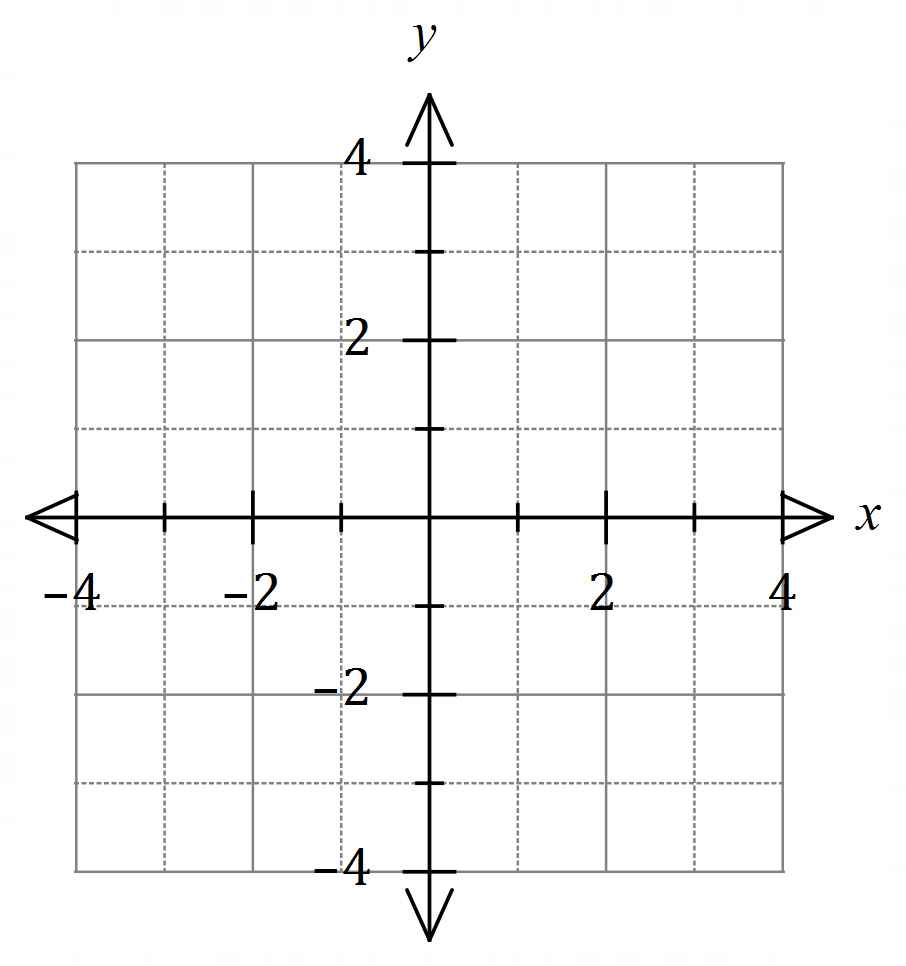
Question 13 (9 marks)

(a) Sketch the locus of the complex number given by

(i) . (3 marks)



(ii) . (3 marks)

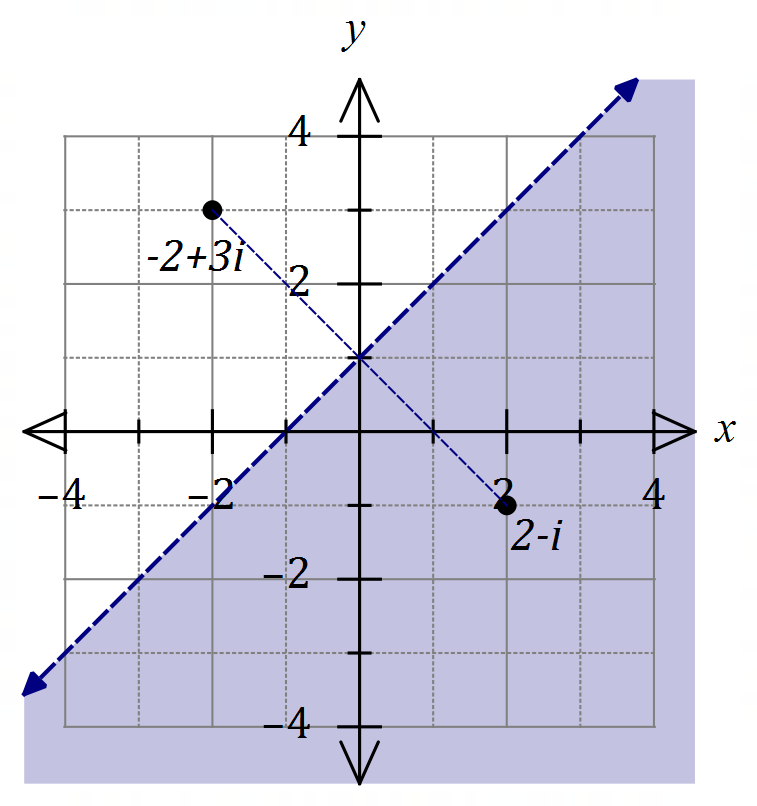


(b) For the locus determine, correct to the nearest degree, the minimum value of , . (3 marks)

Question 13 (9 marks)

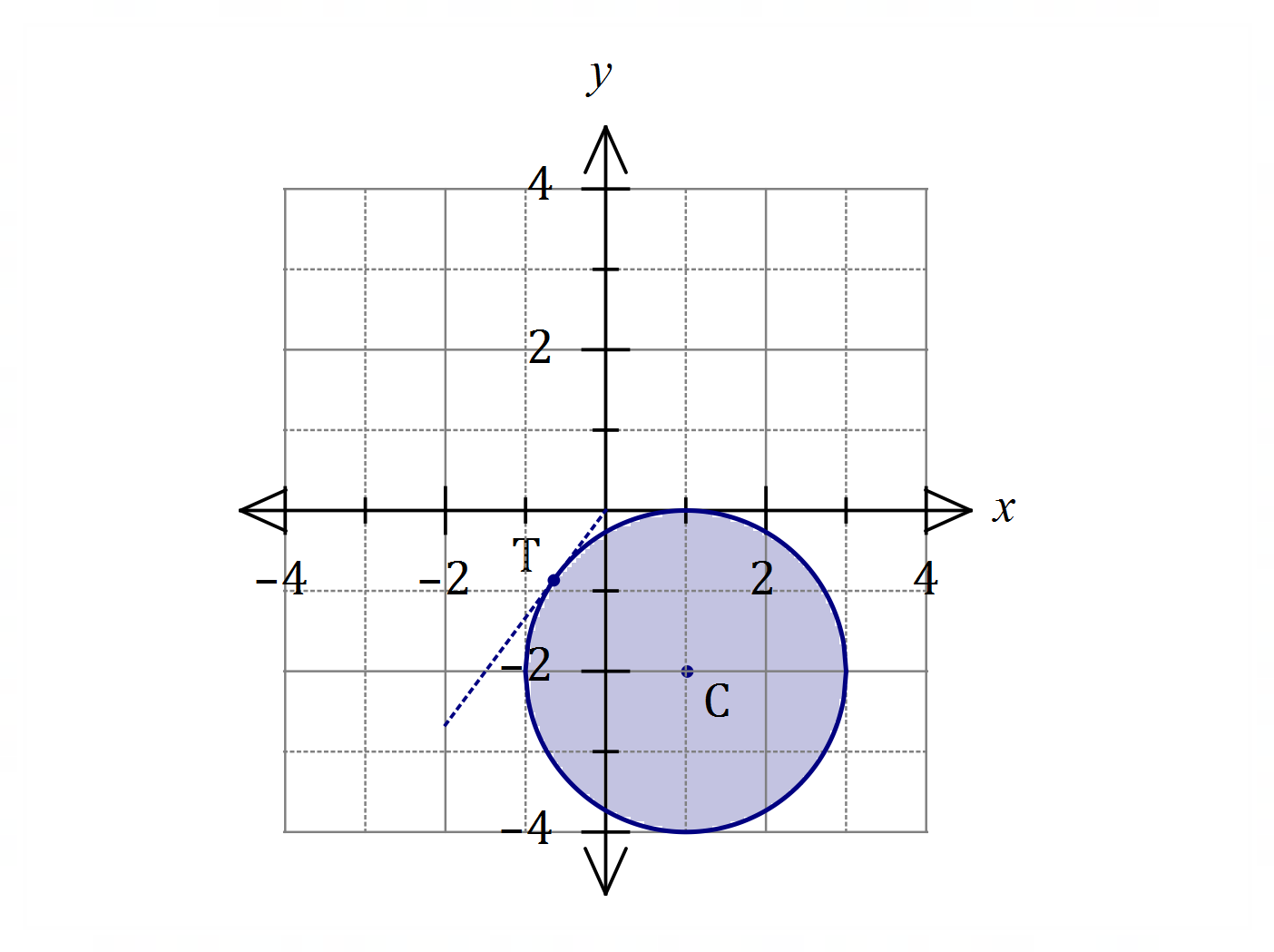
(a) Sketch the locus of the complex number given by

(i) . (3 marks)



|  |
| --- |
| **Solution** |
| See diagram |
| **Specific behaviours** |
| ✓ indicates two points   correct location of bisector   dotted line and correct shading |

(ii) . (3 marks)



|  |
| --- |
| **Solution** |
| See diagram |
| **Specific behaviours** |
| ✓ indicates centre   circle, radius   solid line and shades inside |

(b) For the locus determine, correct to the nearest degree, the minimum value of , . (3 marks)

|  |
| --- |
| **Solution** |
| Minimum argument is angle tangent (shown on diagram) makes with positive -axis.  Using symmetry, minimum |
| **Specific behaviours** |
| ✓ indicates ray on diagram   calculates angle in triangle   correct argument |

Question 18 (8 marks)

Let and .

(a) Solve the equation , giving all solutions in the form .

(3 marks)

(b) Express in both Cartesian and polar forms, where . (2 marks)

(c) Show how to use your answers from part (b) to determine an exact value for

(i) . (2 marks)

(ii) . (1 mark)

Question 18 (8 marks)

Let and .

(a) Solve the equation , giving all solutions in the form .

(3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expresses in polar form   one correct solution   all correct solutions |

(b) Express in both Cartesian and polar forms, where . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct Cartesian form   correct polar form |

(c) Show how to use your answers from part (b) to determine an exact value for

|  |
| --- |
| **Solution** |
| Equating imaginary parts: |
| **Specific behaviours** |
| ✓ equates imaginary parts of in both forms   shows division by modulus to obtain value |

(i) . (2 marks)

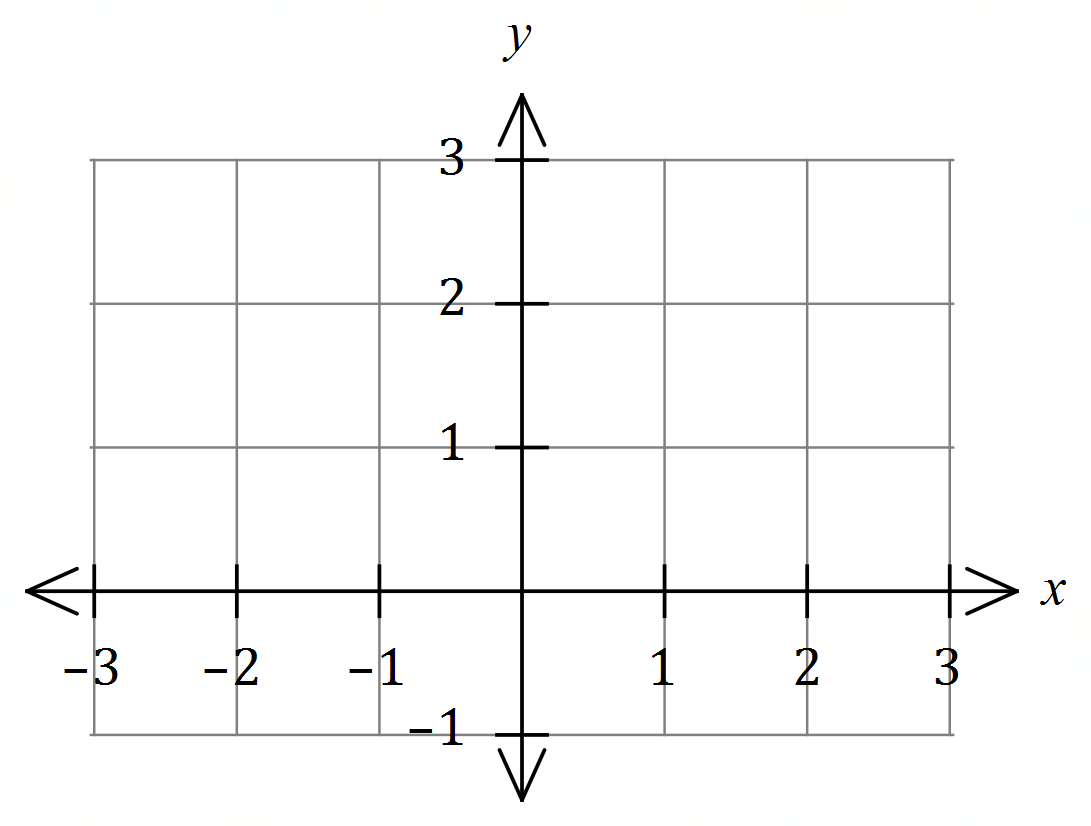
(ii) . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
|  indicates quotient of Im and Re parts to obtain value |

Question 20 (8 marks)

Consider the function , where and .

(a) Determine when and use the Argand diagram below to show that is a reflection of in the line . (3 marks)



Any reflection of in the complex plane can be expressed in the form , where and are complex constants.

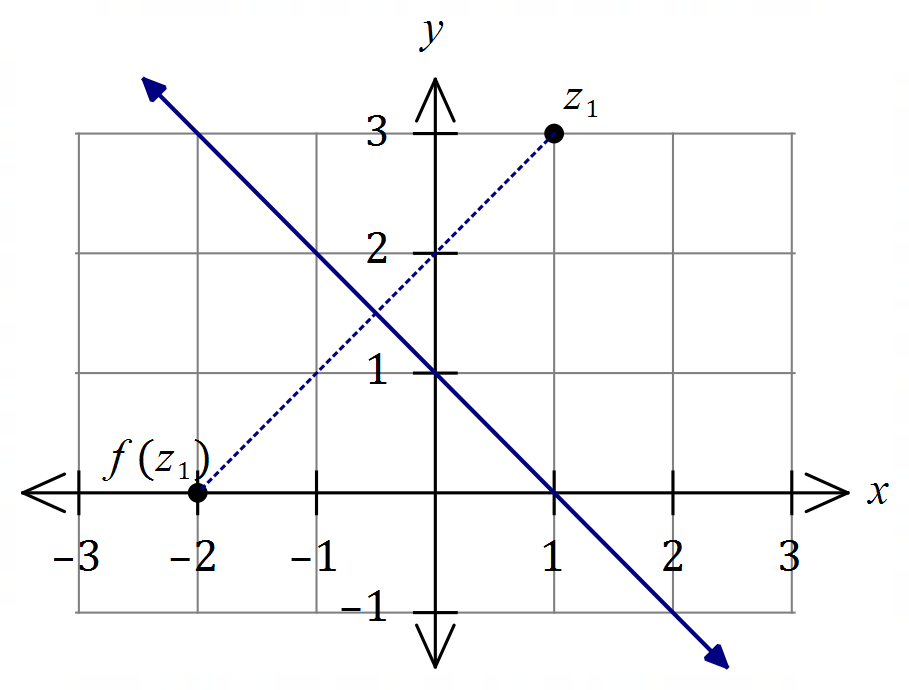
(b) By considering the transformations of the axes intercepts of the reflection line, or otherwise, determine the value of and the value of so that represents a reflection of in the line . (4 marks)

(c) Given , determine , the reflection of in the line . (1 mark)

Question 20 (8 marks)

Consider the function , where and .

(a) Determine when and use the Argand diagram below to show that is a reflection of in the line . (3 marks)



|  |
| --- |
| **Solution** |
| See diagram |
| **Specific behaviours** |
| ✓ evaluates function   plots and   line, indicates reflection |

Any reflection of in the complex plane can be expressed in the form , where and are complex constants.

(b) By considering the transformations of the axes intercepts of the reflection line, or otherwise, determine the value of and the value of so that represents a reflection of in the line . (4 marks)

|  |
| --- |
| **Solution** |
| Intercepts: and .  and .  If a point lies on mirror line, then .  and  Solving simultaneously gives |
| **Specific behaviours** |
| ✓ intercepts   forms two equations in and   solves one constant   solves both constants |

(c) Given , determine , the reflection of in the line . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct complex number |

Question 9 (4 marks)

Solve the equation , giving solutions in polar form where and .

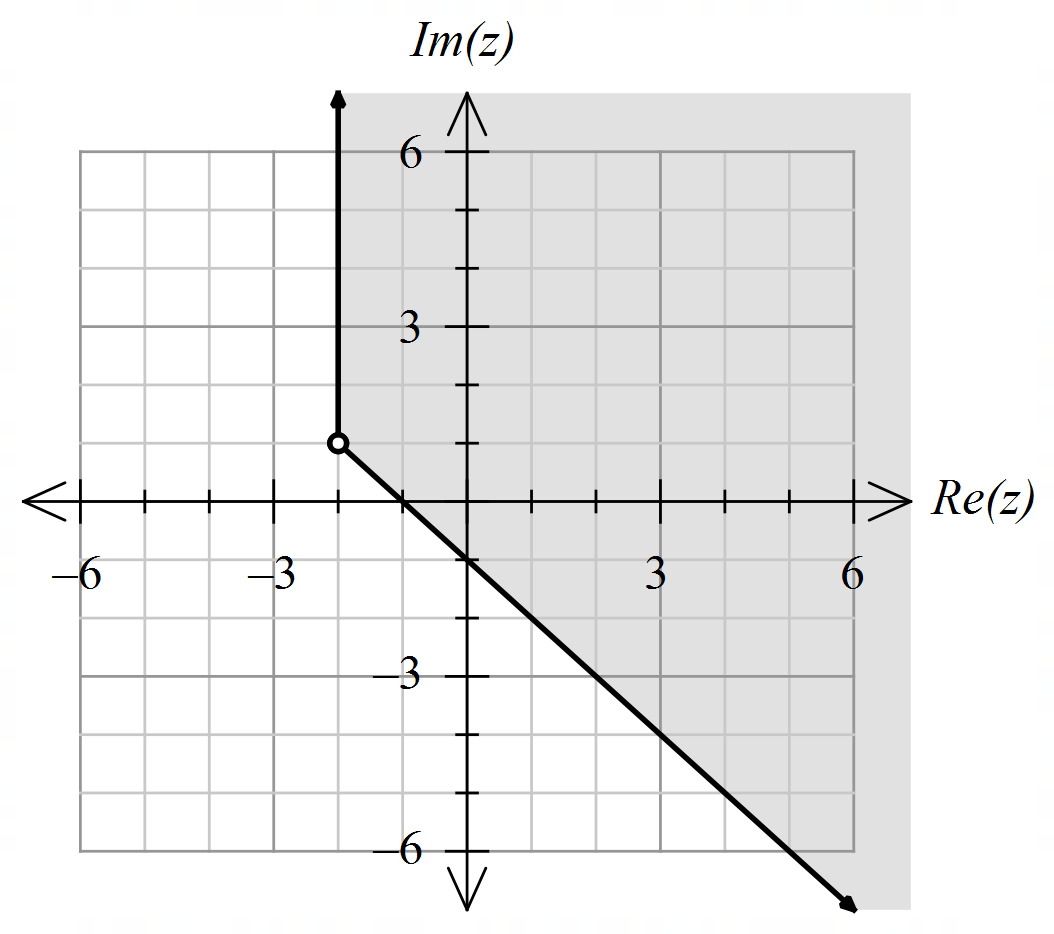
Question 9 (4 marks)

Solve the equation , giving solutions in polar form where and .

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ converts to polar form   applies de Moivre's   one correct root   all correct roots |

Question 10 (4 marks)

The locus of a complex number is shown below.

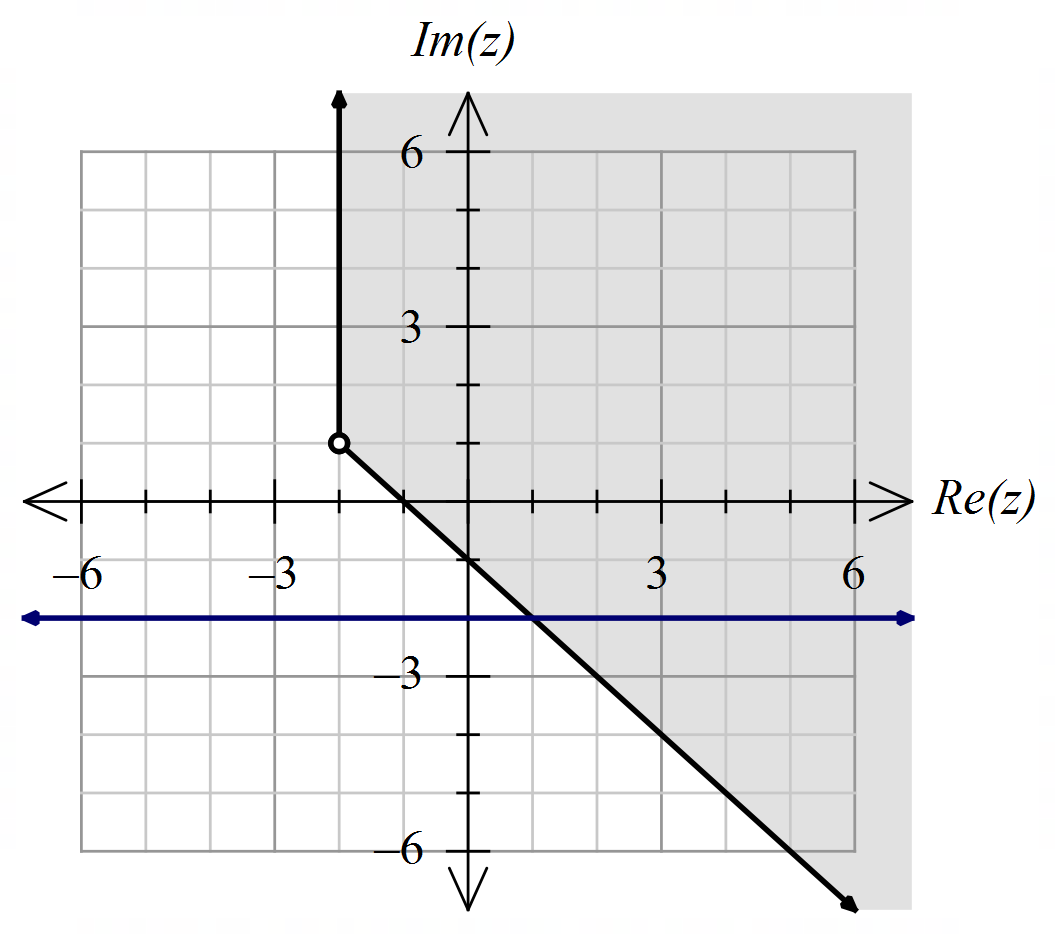


(a) Without using or , write an inequality in terms of for the locus. (3 marks)

(b) Add the locus for to the diagram above. (1 mark)

Question 10 (4 marks)

The locus of a complex number is shown below.



|  |
| --- |
| **Solution (b)** |
| See diagram |
| **Specific behaviours** |
| ✓ correct line |

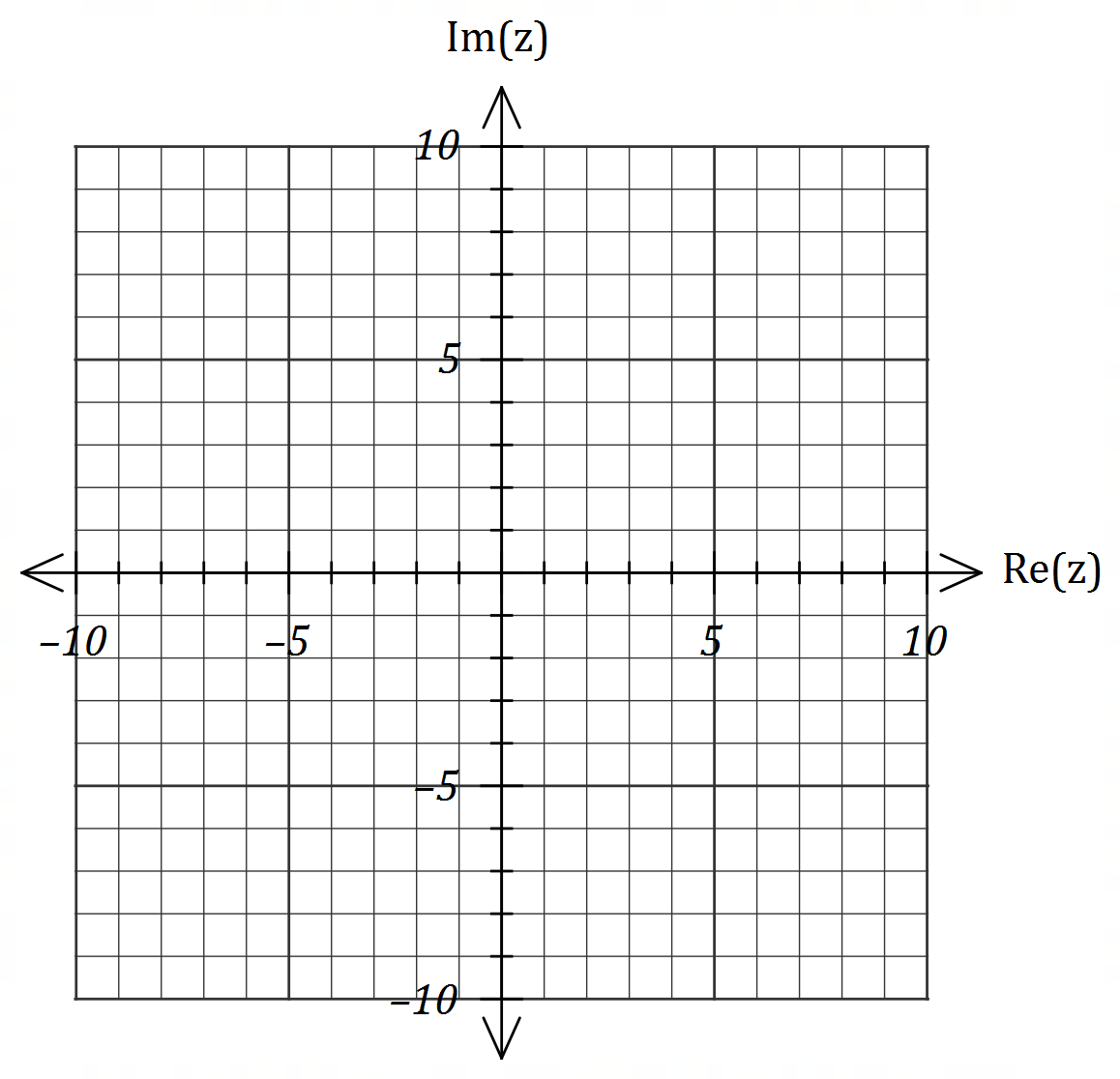
(a) Without using or , write an inequality in terms of for the locus. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ lower argument   upper argument   translation |

(b) Add the locus for to the diagram above. (1 mark)

Question 17 (8 marks)

(a) Indicate the subset of points in the complex plane that satisfy on the axes below. (3 marks)



(b) Given that , determine

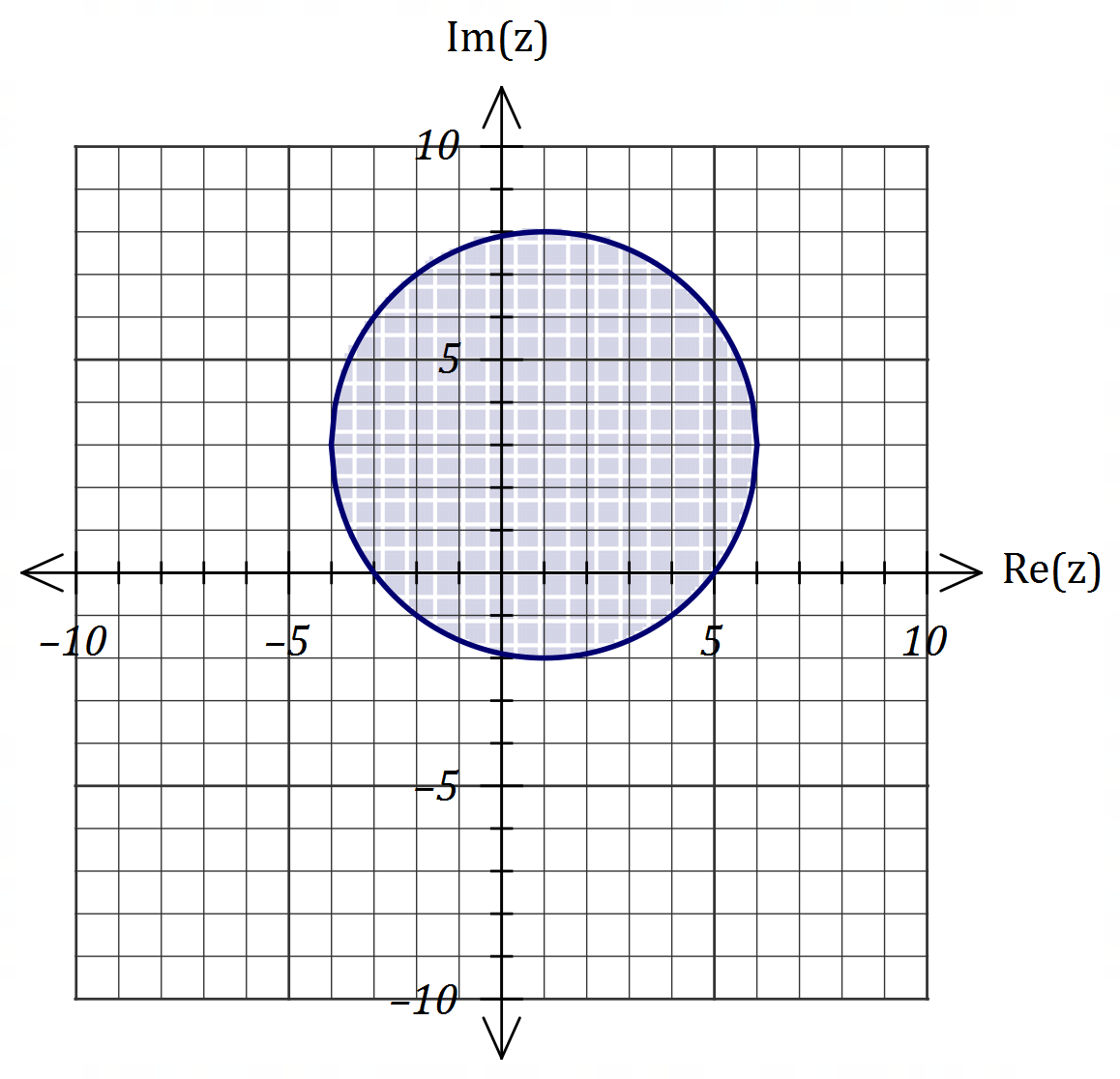
(i) the minimum value of . (1 mark)

(ii) the maximum value of . (2 marks)

(iii) the maximum value of . (2 marks)

Question 17 (8 marks)

(a) Indicate the subset of points in the complex plane that satisfy on the axes below. (3 marks)



|  |
| --- |
| **Solution** |
| On and inside circle with centre and . |
| **Specific behaviours** |
| ✓ circle, correct centre   correct radius   shades region |

(b) Given that , determine

(i) the minimum value of . (1 mark)

|  |
| --- |
| **Solution** |
| Minimum is . |
| **Specific behaviours** |
| ✓ correct value |

(ii) the maximum value of . (2 marks)

|  |
| --- |
| **Solution** |
| Require maximum distance from to point on circle - will lie on ray from centre. |
| **Specific behaviours** |
| ✓ indicates correct method   correct value |

(iii) the maximum value of . (2 marks)

|  |
| --- |
| **Solution** |
| Hence maximum value is . |
| **Specific behaviours** |
| ✓ indicates correct method   correct value |

Question 21 (6 marks)

(a) Determine the cube roots of , giving roots in polar form where .

(3 marks)

(b) One of the cube roots of is also a fourth root of .

If is the argument of a fourth root of that lies in the first quadrant , determine all possible values of . (3 marks)

Question 21 (6 marks)

(a) Determine the cube roots of , giving roots in polar form where .

(3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expresses in polar form   one correct root   all 3 roots |

(b) One of the cube roots of is also a fourth root of .

If is the argument of a fourth root of that lies in the first quadrant , determine all possible values of . (3 marks)

|  |
| --- |
| **Solution** |
| has four roots evenly spaced at , one of which is either . |
| **Specific behaviours** |
| ✓ sketch of cube roots   one correct value   all possible values |

Question 21 (7 marks)

(a) Sketch on an Argand diagram the locus of the complex number given by .

(1 mark)

(b) A circle , of radius 6, has its centre lying on and just touches the line .

(i) Draw on your diagram above. (2 marks)

(ii) Determine the equation of in the form . (2 marks)

(iii) The complex number lies on . Determine the minimum value of , where . (2 marks)

Question 21 (7 marks)

(a) Sketch on an Argand diagram the locus of the complex number given by .

(1 mark)

|  |
| --- |
| **Solution (a), (b)(i)** |
|  |
| **Specific behaviours** |
| ✓ (a) line segment with gradient  ✓ (b)(i) centre on  ✓ (b)(i) touching axis |

(b) A circle , of radius 6, has its centre lying on and just touches the line .

(i) Draw on your diagram above. (2 marks)

(ii) Determine the equation of in the form . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ -coord  ✓ correct equation |

(iii) The complex number lies on . Determine the minimum value of , where . (2 marks)

|  |
| --- |
| **Solution** |
| Minimum |
| **Specific behaviours** |
| ✓ indicates congruent triangles  ✓ correct minimum value |